Scale Invariance and Hierarchy in National Road Networks

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Abstract Networks

Edge cost not a function of spatial dimensions, e.g.,

- World Wide Web
- Email
- Interaction networks
  - proteins, genes, cell-signals, etc.
  - species (predator-prey, host-parasite)
- Social networks (sort of)
Spatial Networks

Edge cost is function of spatial separation, e.g.,

- Communication
  - wired and wireless transmission
- Transportation
  - distributing resources or goods (pipes)
  - travel (roads)
Road Networks

- We study national road networks of the USA, England and Denmark
- Via their dual representation, we show that
  - they (intuitively) have hierarchical organization
  - dual degree distribution is a power law
  - journeys exhibit scale invariant structure
  - these patterns agree with a fractal placement of roads and intersections
The Dual network

- Each “named” road is a vertex
- Two vertices linked if their roads ever intersect
Dual Road network

Path on dual graph corresponds to driving directions:

- Start on 15th St.
- Turn right onto Indiana Ave.
- Left on 7th St.
- Stop at Union Building
Network Sampling

• We sample the dual graph by querying a commercial service (e.g., Google Maps) for driving directions
• Source and destinations: uniformly random postal-code pairs
• Note: this under samples roads within postal codes, but, local roads already well-studied
**Dual Degree Distributions**

- Power-law degree distribution for all three nations
- largest degree roads are national highways (many interchanges)
- no universal scaling exponent; value depends on fractal distributions (more later)

<table>
<thead>
<tr>
<th>Network</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>2.4</td>
</tr>
<tr>
<td>England</td>
<td>2.2</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.4</td>
</tr>
</tbody>
</table>
Dual Degree Distributions

- United States: $\alpha = 2.4$
- England: $\alpha = 2.2$
- Denmark: $\alpha = 2.4$
Journey Structure (1)

- Define **profile**: largest step (centered), flanked by three largest preceding and subsequent steps (in order).

- Journey profile is scale invariant over short, medium and long journeys, for all three nations.

- Conjecture: This is true for all sufficiently large national road networks.
Journey Structure (1)
Journey Structure (2)

- Journey structure itself is scale invariant: the fraction of total distance covered by $k^{th}$ step is constant and independent.

<table>
<thead>
<tr>
<th>Network</th>
<th>Distance</th>
<th>1st largest</th>
<th>2nd largest</th>
<th>3rd largest</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0-750</td>
<td>0.460±</td>
<td>0.212±</td>
<td>0.118±</td>
</tr>
<tr>
<td></td>
<td>751-1250</td>
<td>0.397±</td>
<td>0.208±</td>
<td>0.129±</td>
</tr>
<tr>
<td></td>
<td>1251+</td>
<td>0.382±</td>
<td>0.201±</td>
<td>0.132±</td>
</tr>
</tbody>
</table>

- Suggests scaling relationship $s_j = A_j \ell$
fit data with
\[ s_j = A_j \ell^{\alpha_j} \]
and find that
\[ \alpha_j = 1.0 \pm 0.1 \]
Fractal Structure

• Define: $d_p$, the fractal dimension for distribution of intersections in plane

• Define: $d_i$, the fractal dimension for distribution of intersections on a single road

• If road placement follows this fractal structure, the dual degree distribution scales like

$$\alpha = 1 + \frac{d_p}{d_i} \quad \text{(see paper)}$$
## Fractal Structure

<table>
<thead>
<tr>
<th>Schema</th>
<th>$d_p$</th>
<th>$d_i$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>$\log_3 9$</td>
<td>$\log_3 3$</td>
<td>3.00</td>
</tr>
<tr>
<td>all but center</td>
<td>$\log_3 8$</td>
<td>$\log_3 3$</td>
<td>2.89     (Sierpinski)</td>
</tr>
<tr>
<td>odd numers</td>
<td>$\log_3 5$</td>
<td>$\log_3 2$</td>
<td>3.32</td>
</tr>
<tr>
<td>corners</td>
<td>$\log_3 4$</td>
<td>$\log_3 2$</td>
<td>3.00</td>
</tr>
</tbody>
</table>
Challenges

- Adapt fractal model to generate more statistically realistic road network
  - e.g., incorporate traffic densities, population densities, etc.
- Validate connection between $\alpha$ and $d_p, d_i$; measure real fractal dimensions
- Comparison with other transportation networks, e.g., natural gas distribution pipelines, circulatory system

Fin