IPD Network Creation Games

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Motivation

- Optimization of network topologies is often computational expensive
- Distributive alternative:
  Game Theory coupled to network topology

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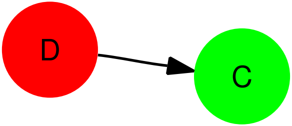
Outline

Prisoner's Dilemma (PD)

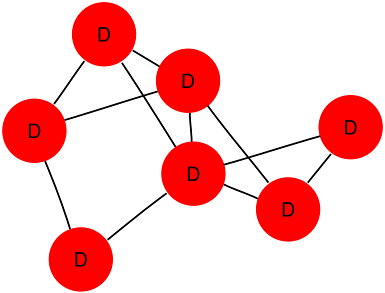


- Two nodes / players
- Strategies: Cooperate, Defect
- Payoff Matrix $\pi = \begin{pmatrix} P & T \\ S & R \end{pmatrix}$

  P Punishment
  T Temptation to defect
  S Suckers payoff
  R Reward
- $2R > T > R > P > S$
- Common values:
  $R = 3,\ P = 1,\ S = 0$

Nash Equilibrium

- Egoistic players
- Every player with optimal strategy
- No single player can improve its average payoff



- Only 1 Nash Equilibrium in Prisoner's Dilemma
- All players use strategy "D"
- Independent of network structure

Iterated Prisoner's Dilemma (IPD)

How to

- defend yourself against Defectors
- but still work together with Cooperators?

Introduce memory

- Nodes play Prisoner's Dilemma repeatedly
- Each node remembers the opponents last m moves and adjusts its strategy accordingly
- We use $m = 1$ from now on
- $m = 1$ results in eight possible strategies

IPD Strategies

?,C,D	Binary	#	Name
D,D,D	000	0	Always Defect
D,D,C	001	1	Suspicious Anti Tit-for-Tat
D,C,D	010	2	Suspicious Tit-for-Tat
D,C,C	011	3	Suspicious Cooperate
C,D,D	100	4	Generous Defect
C,D,C	101	5	Generous Anti Tit-for-Tat
C,C,D	110	6	Generous Tit-for-Tat
C,C,C	111	7	Always Cooperate

8×8 payoff matrix of the IPD

$$
\begin{pmatrix}
P & T & P & T & P & T & P & T \\
S & \frac{P+R}{2} & \frac{P+S+R+T}{4} & T & S & T & \frac{T+P+S+R}{4} & T \\
P & \frac{P+T+R+S}{4} & P & R & P & \frac{T+R+S+P}{4} & \frac{T+S}{2} & R \\
S & S & R & R & S & S & R & R \\
P & T & P & T & P & T & P & T \\
S & S & \frac{S+R+T+P}{4} & T & S & \frac{R+P}{2} & \frac{R+T+P+S}{4} & T \\
P & \frac{S+P+T+R}{4} & \frac{S+T}{2} & R & P & \frac{R+S+P+T}{4} & R & R \\
S & S & R & R & S & S & R & R
\end{pmatrix}
$$

Outline

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Network Nash Equilibrium

Definition

- Egoistic players
- No single player can improve its payoff . . .
  - by changing its strategy
  - by changing its neighbourhood

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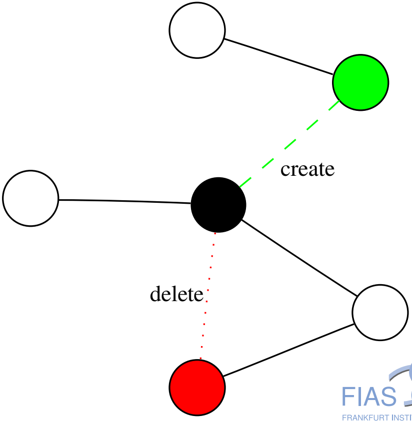
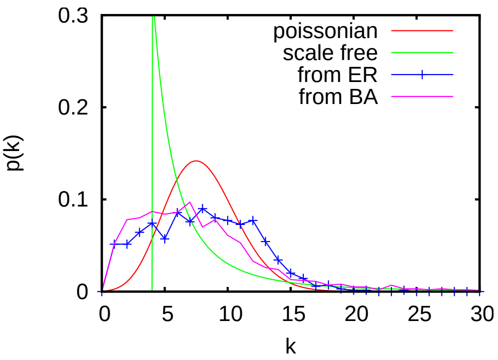
New Node Encounter

- find the player that gives highest payoff
- create a link to that player
- remove the link that yields lowest score



Convergence



- $T = 4.0$
- 1. Network Nash Equilibrium
- 100. Network Nash Equilibrium
- 1000+. Network Nash Equilibrium

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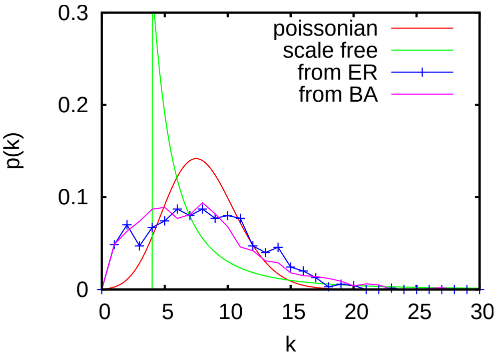
Convergence



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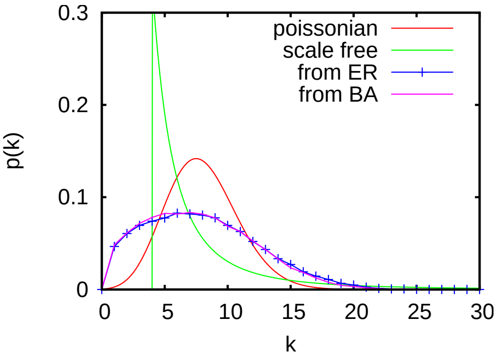
Convergence



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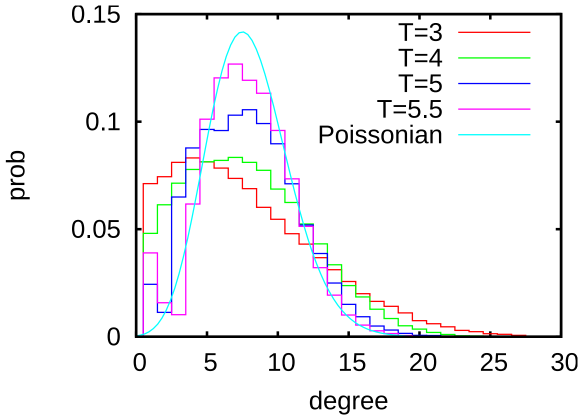
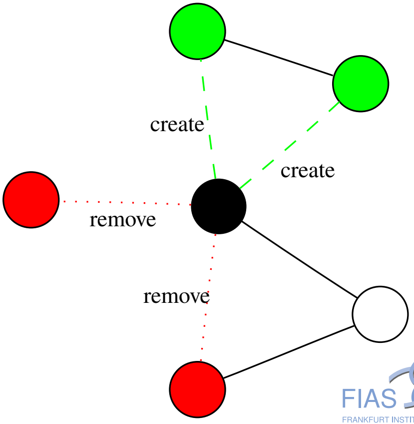
T-Dependence of Node Degree

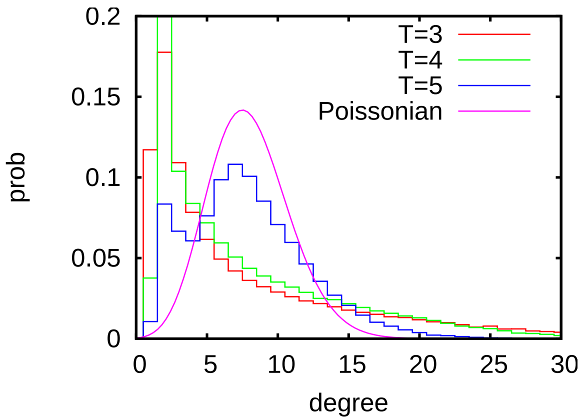


New Link Encounter

- find the pair of players that gives highest payoff
- create links to those players
- remove the two links that score worst



T-Dependence of Node Degree



Outline

1. Game Theory
   - Prisoner's Dilemma and Nash Equilibrium
   - Iterated Prisoner's Dilemma
2. Coupling of IPD to Network Structure
   - Network Nash Equilibrium
   - New Node Encounter
   - New Link Encounter
3. Benchmarking
   - Benchmark Function
   - Results

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Centrality vs. low Degree

Nodes in communication networks want:

- short distances to other nodes:
  large closeness centrality $c_i = \sum_{j=1(j \neq i)}^{N} d_{ij}^{-1}$
- small investment:
  low degree k_i

$$E = \frac{1}{N(N-1)} \sum_{i=1}^{N} \frac{c_i}{k_i}$$

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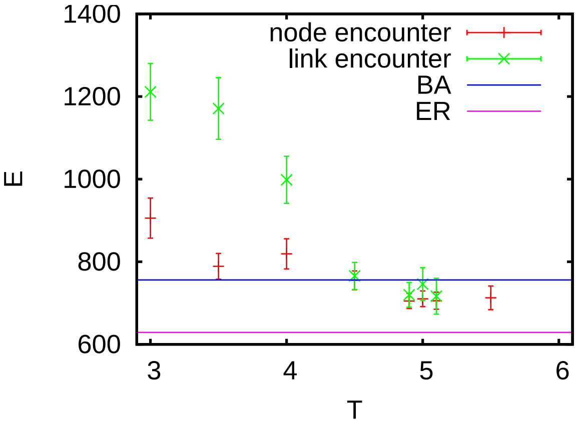
Benchmarking the Emerging Networks



$$E \sim \sum_{i=1}^{N} \frac{c_i}{k_i}$$

Summary

- Game Theory provides a decentralized way to modify network topologies
- Emergent network structure depends on payoff matrix
- Coupling of game to network structure *can* optimize the network with respect to some benchmark