When Network Eigenvector Centrality Misbehaves: Some Lessons

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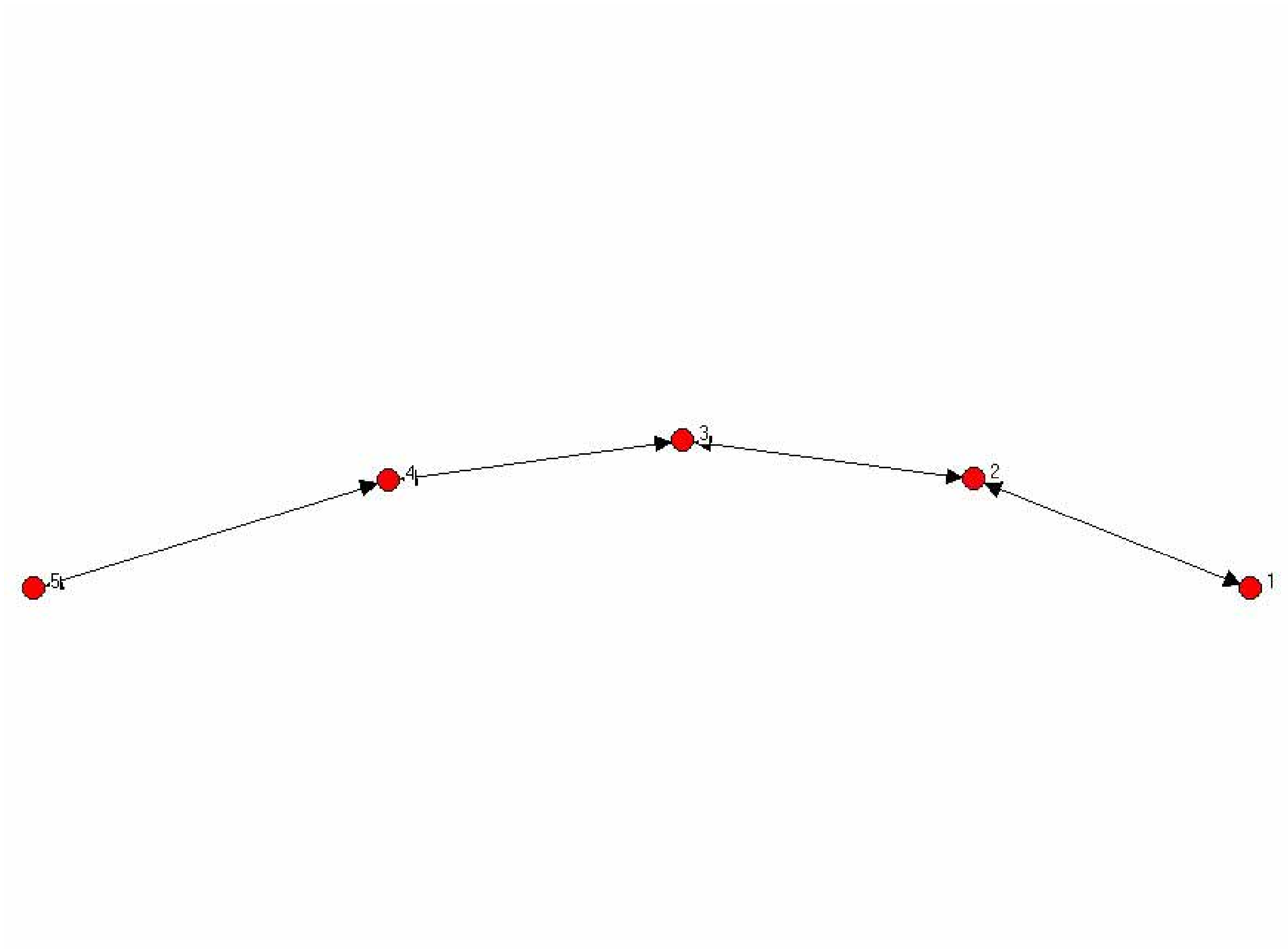
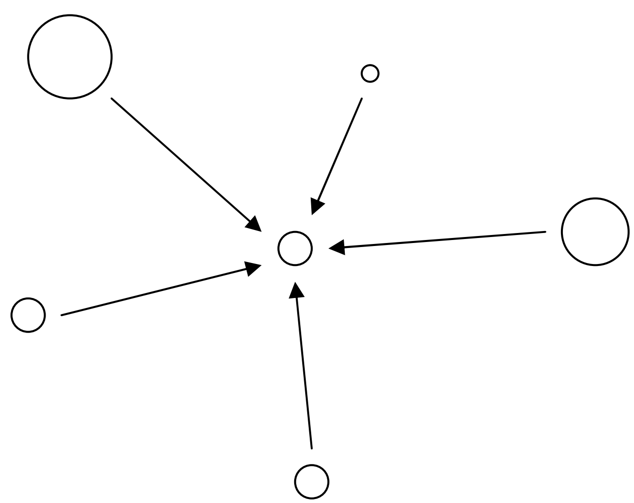
Simple illustrative network



Eigenvector Centrality

$$\lambda x_j = a_{1j}x_i + a_{2j}x_2 + ... + a_{nj}x_n$$

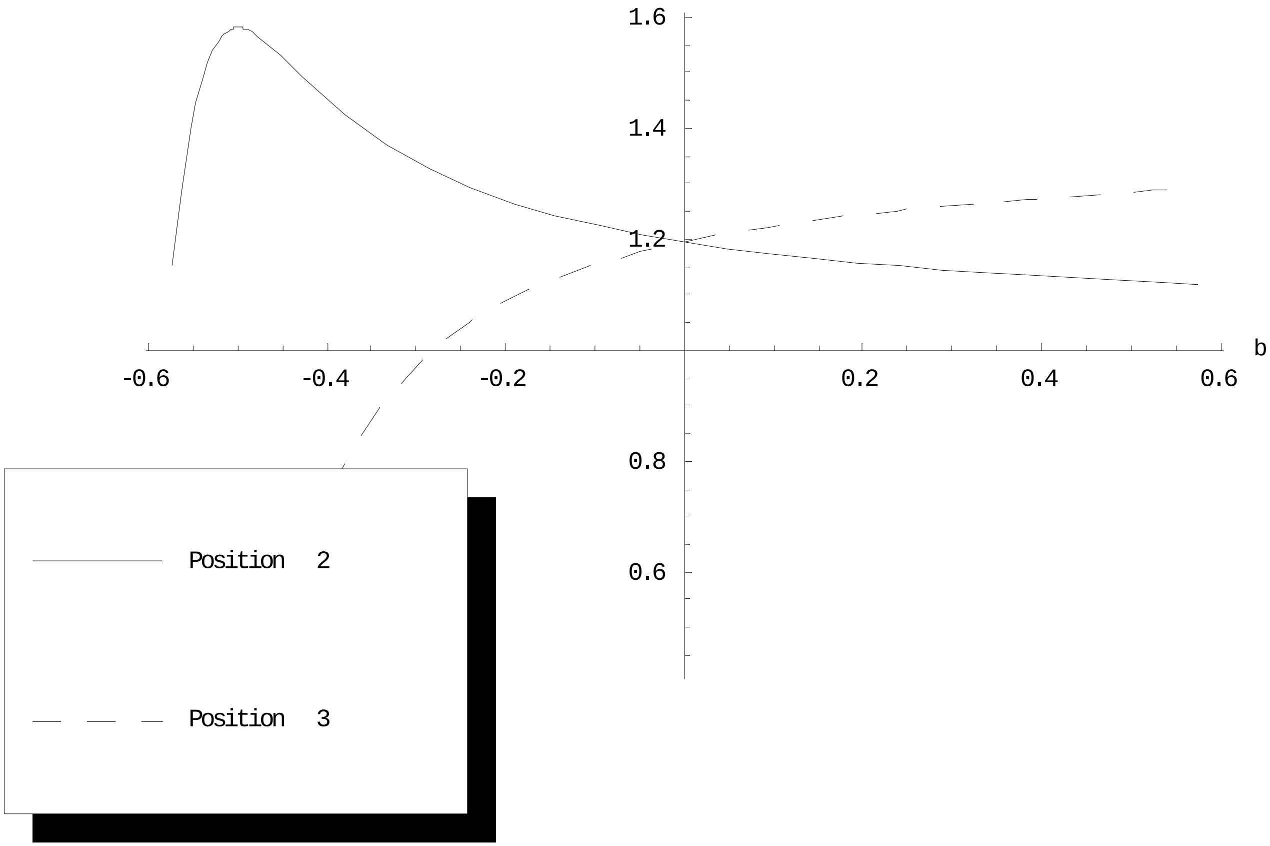


The equations

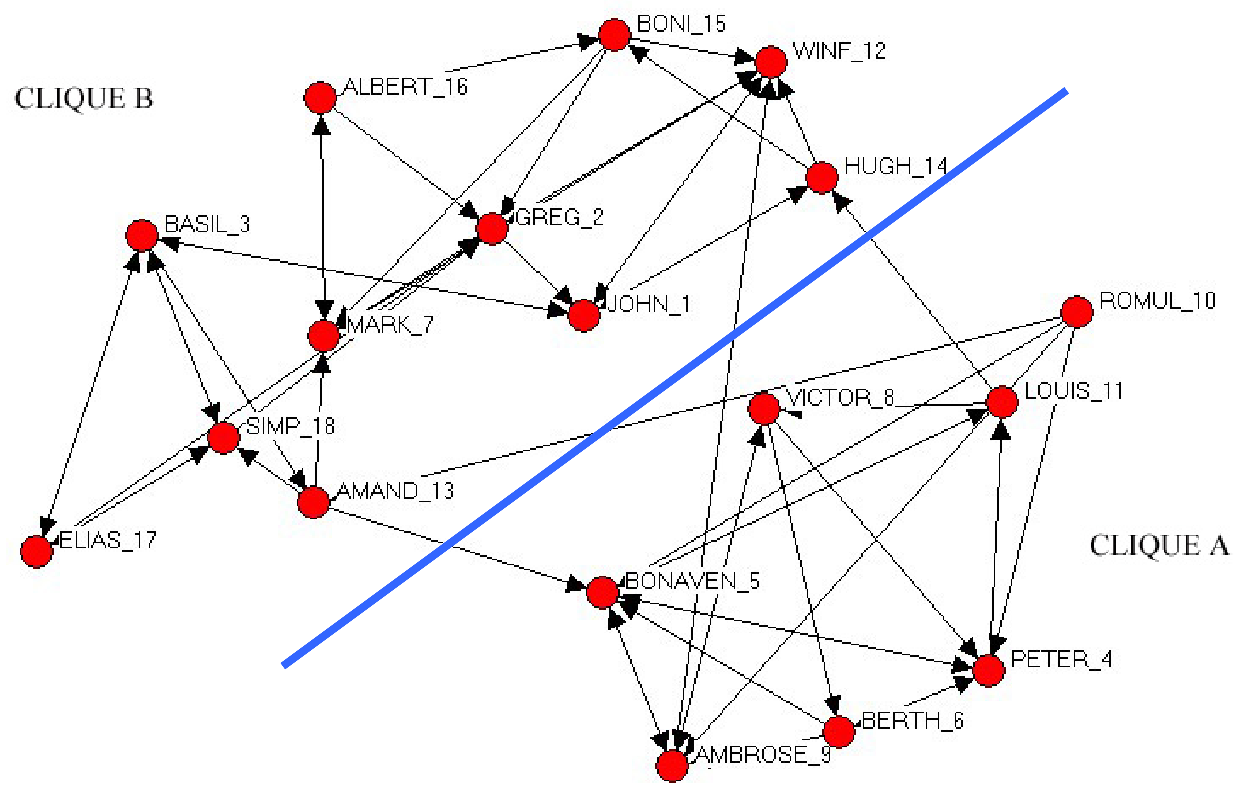
$$c(\beta) = (I - \beta A)^{-1} A\mathbf{1} = \sum_{i=1}^{\infty} \beta^{i-1} A^{i} \mathbf{1}$$

$$\lim_{\beta \to \frac{1}{\lambda}^-} c(\beta) = x$$

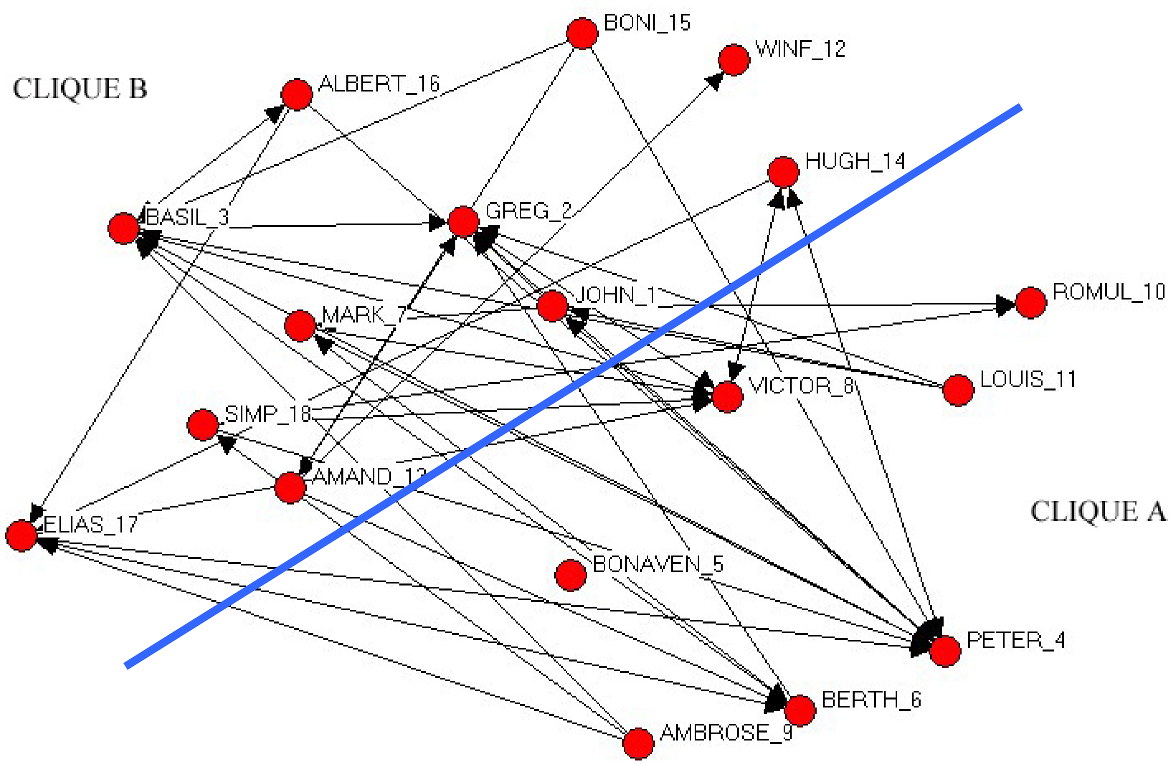
The effect of variations in ♌ on c(♌)



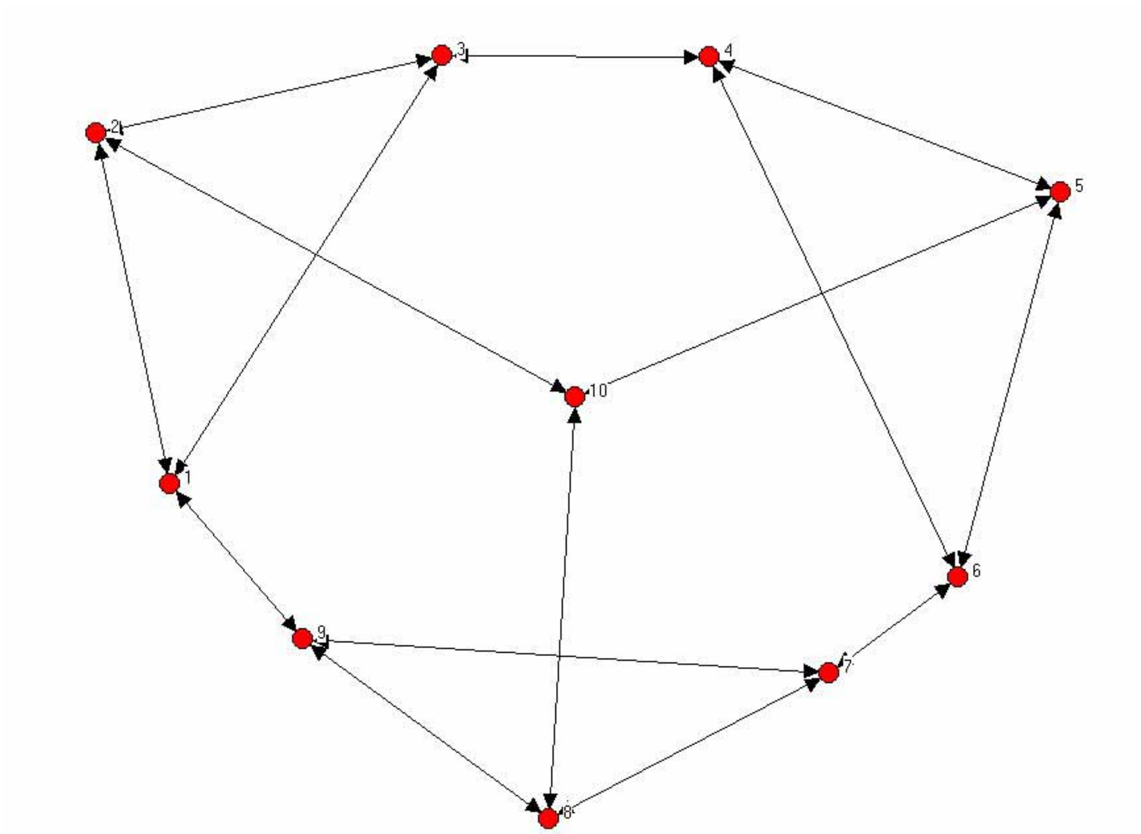
Positive ties in Sampson's monastery



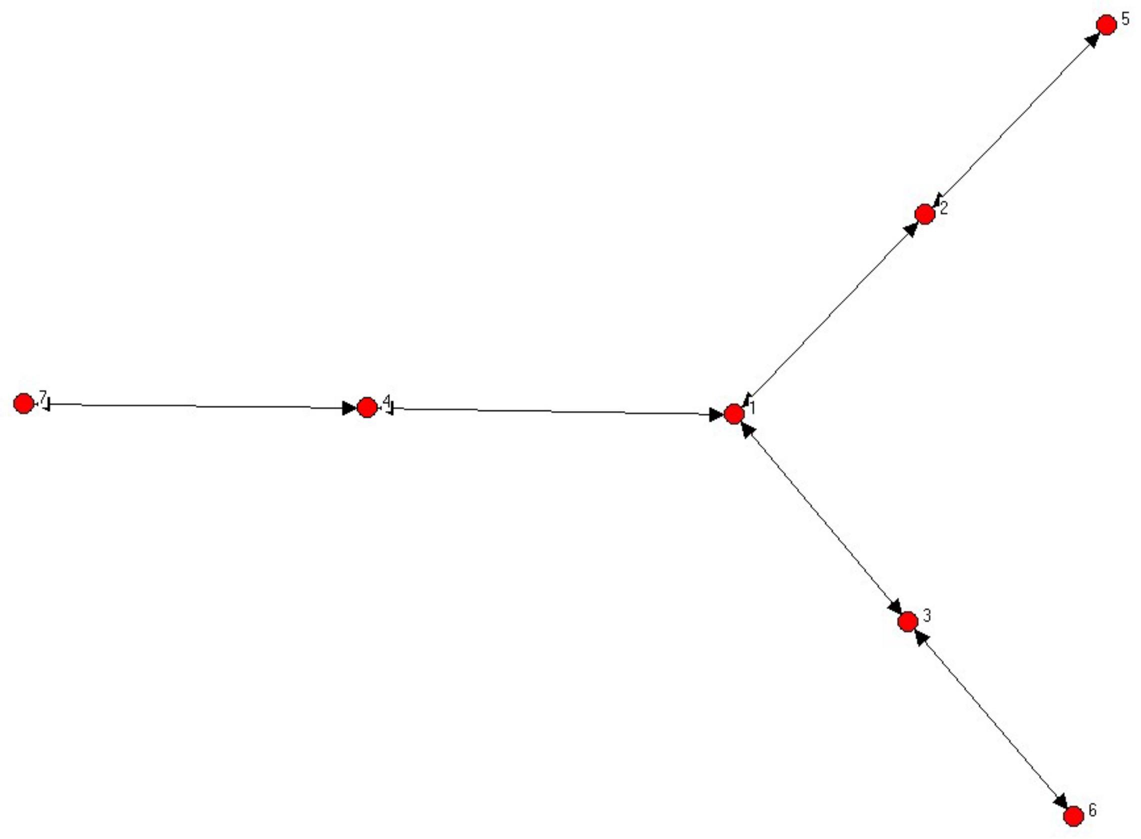
Negative ties in Sampson's monastery



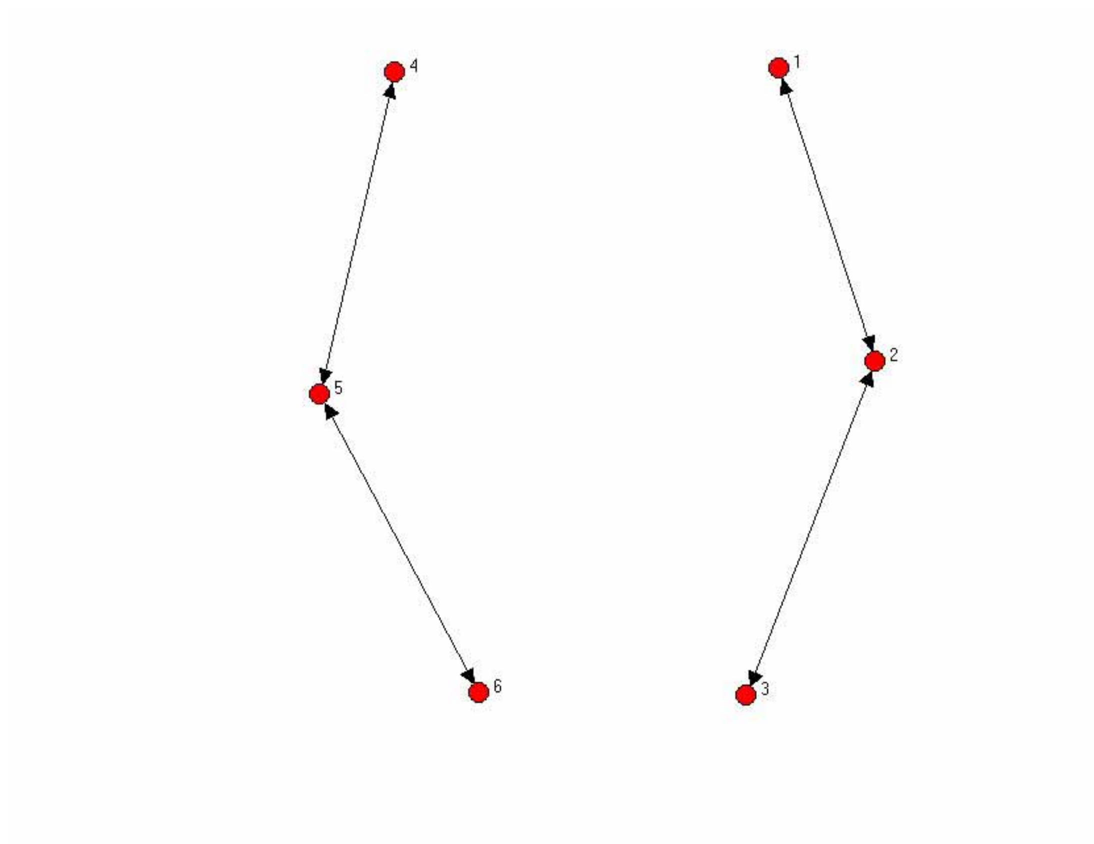
A regular network. Eigenvector centrality fails to distinguish between vertices



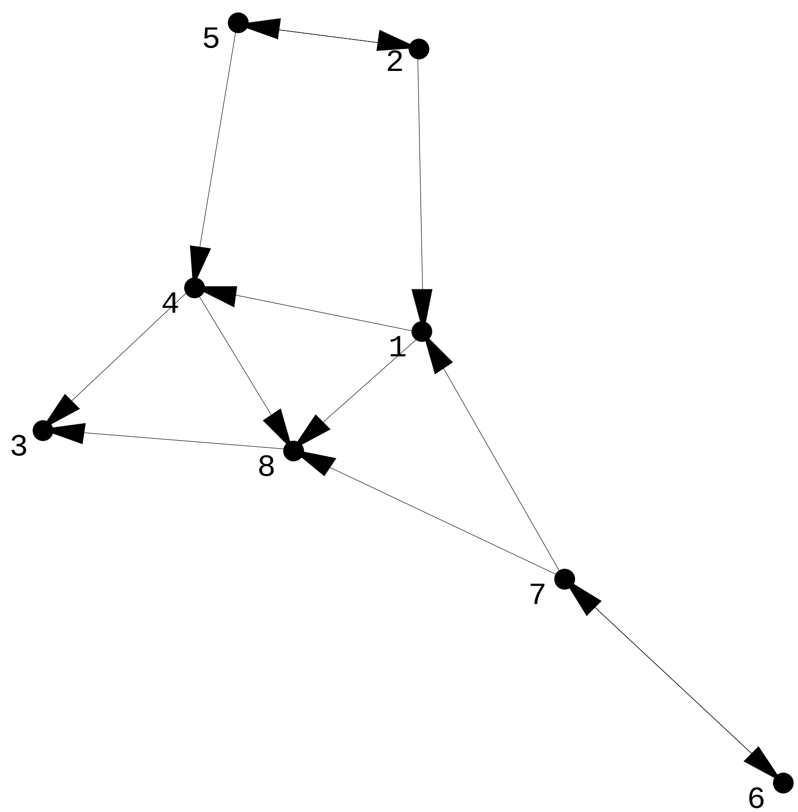
Eigenvector centrality no better than degree because $A1 = cA^2 1$



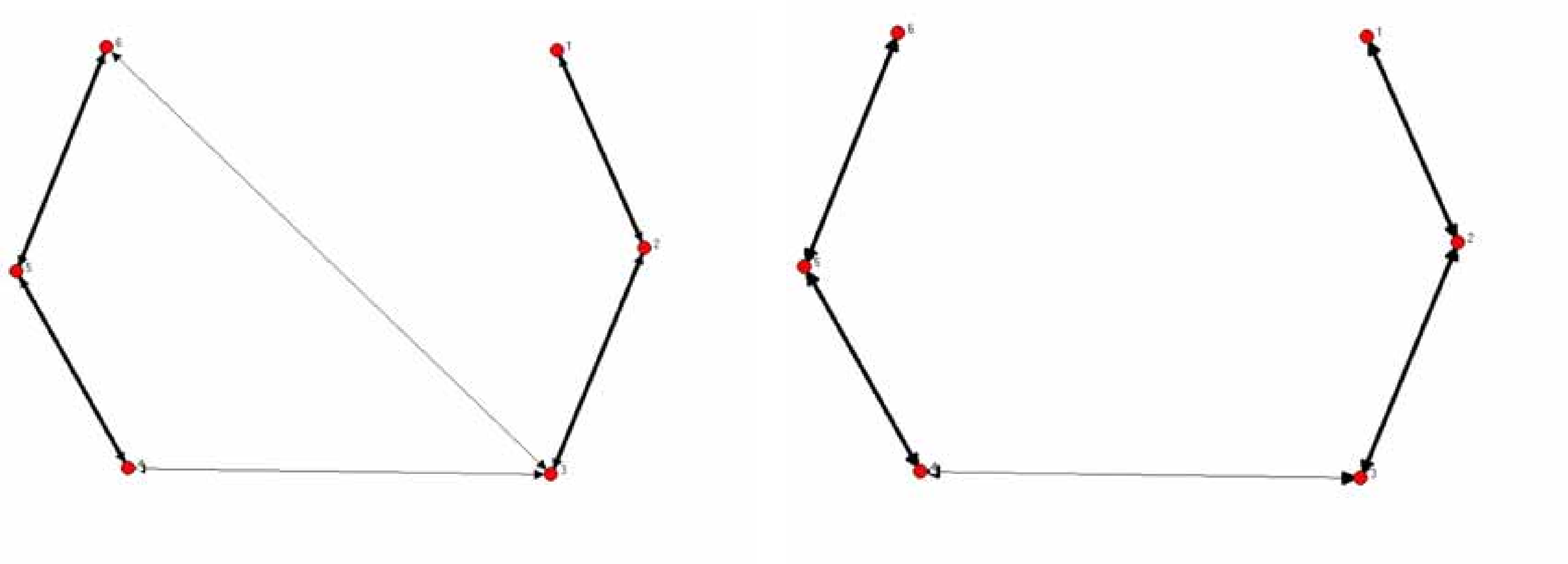
C(♌) does not approach an eigenvector because $\bullet_1 = \bullet_2$



A digraph in which c(♌) does not approach an eigenvector



Two balanced networks



When $x_1 1=0$, $c(\beta)$ does not approach x_1

$$c(\beta) = (I - \beta A)^{-1} A1$$

$$= (A + \beta A^2 + \beta^2 A^3 \ldots)1 \ when \left|\beta\right| < \tfrac{1}{\lambda_1}$$

$$A = \sum_{i=1}^{n} \lambda_i x_i x_i^t \ \ where\ x_i^t x_i = 1\ and\ \ x_i^t x_j = 0\ \ for\ i \neq j\ because\ A\ is\ symmetric.$$

$$Therefore,\ A^k = \sum_{i=1}^{n} \lambda_i^k x_i x_i^t \ \ and \quad \beta^{k-1} A^k = \frac{1}{\beta} \sum_{i=1}^{n} (\beta\lambda_i)^k x_i x_i^t$$

$$And,\ c(\beta) = \sum_{k=1}^{\infty} \beta^{k-1} A^k 1 = \frac{1}{\beta} \sum_{k=1}^{\infty} \sum_{i=1}^{n} (\beta\lambda_i)^k x_i x_i^t 1$$

$$= \frac{1}{\beta} \sum_{i=1}^{n} x_i x_i^t \sum_{k=1}^{\infty} (\beta\lambda_i)^k 1 = \frac{1}{\beta} \sum_{i=1}^{n} x_i x_i^t \frac{\beta\lambda_i}{1 - \beta\lambda_i} 1$$

$$= \sum_{i=1}^{n} \frac{\lambda_i x_i^t 1}{1 - \beta\lambda_i} x_i$$