Scale Invariance and Hierarchy in National Road Networks

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To date, the study of complex networks has focused primarily on topological quantities such as the degree distribution, diameter and clustering coefficient. While virtual networks like the World Wide Web, or interaction networks like that of proteins, may be considered purely in terms of such topology, physical networks have additional geographic properties. In particular, creating and maintaining edges presumably requires physical resources proportional to their length, and the physical length of a path between two vertices may be rather different from its topological length (i.e., the number of edges along it). In some cases, the interaction of a network's topology with its underlying geography has been studied previously through models of evolving networks or optimizing resource costs [1, 2].

Here, we study the presence of hierarchy and scale invariance in physical networks as illustrated by the national road networks of the United States, England and Denmark [3]. To reveal their topological organization, we employ the *dual* model of the road network, in which a vertex represents a single road of a given name, and two vertices are joined if their corresponding roads ever intersect. This should not be confused with the dual of a planar graph, in which faces become vertices and vice versa. This graph transformation has been used previously to study the topological structure of urban roads [4, 5, 6, 7].

Through this representation, we show empirically that the dual degree distribution has a heavy tail and is well-characterized by a power law with an exponent $2.2 \leq \alpha \leq 2.4$. Additionally, we find that the structure of journeys on the physical network is similar regardless of scale, i.e., they are scale invariant. To explain these properties, we introduce and analyze a simple fractal model for the hierarchical placement of roads on the unit square. We show that the recursive nature of this model generates the scale invariant journey structure, and suggests a simple relationship between the scaling exponent of the dual degree distribution α and the fractal dimensions governing the placement of roads and intersections.

To construct the dual representation, we sampled the national road networks of the United States, England and Denmark by querying a commercial service, provided by Mapquest.com. This service provides driving directions, i.e., a path through the dual graph, between some source and destination as a list of road names, the respective distances a driver should travel on each, and instructions as to how to get from one road to another, e.g., "turn left onto" or "continue on". Our sampled networks were constructed from the directions returned for connecting a large number of pairs of postal codes, selected uniformly at random. Notably, our results are biased according to the population distribution, which postal codes roughly follow, and in favor of long journeys rather than more typical short journeys within the same postal code area.

Road networks are intuitively structured in a hierarchical fashion, with minor and major local streets, regional roads, and finally highways. Assuming that a driver wishes to reach her destination as quickly as possible, we may model the structure of an arbitrary journey as follows. Our driver begins at the local street where her point of origin is located, and moves to progressively larger and faster roads, i.e., she moves up the hierarchy, until she reaches the fastest single road between her source and destination. On this road, she covers as much distance as possible, and then descends to progressively smaller roads until she reaches the local street of her destination.

Thus, we expect that the largest steps of a journey will cover a significant fraction of the total distance, and that the length of a step will increase as a driver moves up the hierarchy in the beginning of the journey, and decrease as she descends it at the journey's end. Empirically, we find that this assumption reflects the structure of journeys through our sampled networks. For the purposes of comparison, we classify journeys into three roughly equally populated groups based on their length: short, medium and long. Additionally, we define a journey's *profile* in the following way. We take the largest step of the journey, in terms of distance traveled, the three largest steps (in order of appearance) that precede it, and the three largest steps (again, in order of appearance) that follow it. Thus we ignore the many small steps that are scattered throughout the journey, e.g., taking a highway ramp to merge onto a national highway. While this definition of a journey profile is somewhat arbitrary, it allows us to focus on the journey's large-scale structure.

Figure 1 illustrates the average profile for journeys for the United States network (Denmark and England show sim-

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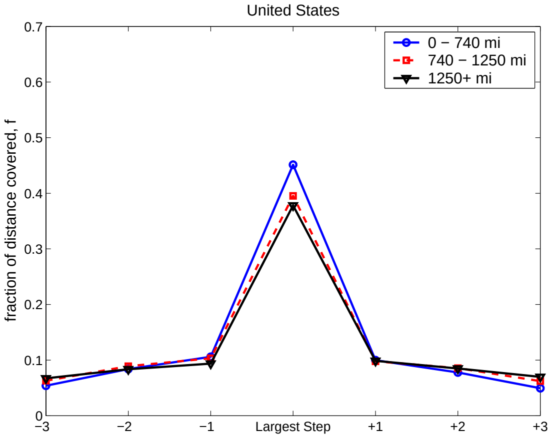

Figure 1: The average journey profile (see text for definition) for the United States.

ilar behavior) for short, medium and long journeys. The unimodal shape of these profiles clearly supports the hierarchical model we describe above, while their approximate collapse across journeys of different lengths indicates that the structure of the journey profile is invariant with respect to the scale of the journey.

By examining the fraction of the total distance covered by the five largest steps of these journeys (see [3] for details), where s_j is the jth largest step, we find that for each j from 1 to 5, the fraction of the journey covered by the jth largest step appears to be roughly constant. This suggests a simple linear relationship of the form

$$s_j = A_j \, \ell \ , \tag{1}$$

where s_j is the jth largest step, ℓ is the total path length and A_j is some constant. Figure 2 shows the average step size for each of the five largest steps against the total path length for the United States network (Denmark and England are similar). We fit our data with a power law of the form $s_j = A_j \, \ell^{\alpha_j}$, bootstrapped via least-squares (we ignore the longest journeys, since we expect finite-size effects to appear as ℓ approaches the diameter of the country). We observe that this power law fits the data reasonably well, with average r^2 values in excess of 0.97; moreover, averaging across all such models, we have $\alpha_j = 1.0 \pm 0.1$, suggesting that the linear form of (1) is accurate.

Finally, we introduce and analyze a simple fractal model for the placement of roads on the unit square that reproduces the observed hierarchical and scale invariant structure of journeys, and the power-law dual degree distribution. Unlike previous models of physical networks, we do not assume an evolving network or an optimization process, but simply take the fractal structure as a given. We show that, under our model, that the scaling exponent of such a network's dual degree distribution is related to the fractal dimensions of the placement of road intersections in the plane (d_p) and along a single road (d_i) in the following way,

$$\alpha = 1 + \frac{d_p}{d_i} \ . \tag{2}$$

The fact that our toy model reproduces the scale invariant journey structure and the correct form of the dual degree distribution suggests that real world road networks may be organized in a fractal fashion. We will also briefly discuss ongoing efforts to validate this hypothesis by analyzing the primal road graph, as well as evocative connections with the allometric scaling hypothesis of biological transport systems [8].

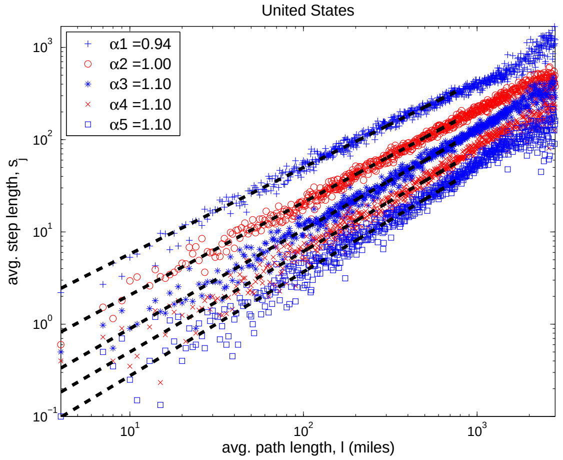

Figure 2: The scale invariant hypothesis predicts that $s_j \approx A_j \, \ell$ for constants A_j, and thus that $\alpha_j \approx 1$. This is consistent with our power-law models. Journeys on the very largest scales were excluded in order to avoid finite-size effects.

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