

Layered Complex Networks

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ABSTRACT

Many complex networks are only a part of larger systems, where a number of coexisting topologies interact and depend on each other. We introduce a layered model to facilitate the description and analysis of such systems. As an example, we study three large real-life transportation systems, where the lower layer is the physical infrastructure and the upper layer represents the traffic flows. First, the layered view allows us to capture fundamental differences between the real load and commonly used load estimators, which explains why these estimators fail to approximate the real load. Next, we study the issue of robustness. We discover that multilayered systems are much more vulnerable to errors and attacks than they seem to be from a single layer perspective.

The layered model was recently introduced in [4]. The analysis of the robustness of layered complex networks is our ongoing work. An algorithm for extracting the two layers from timetables is given in [3].

1. INTRODUCTION

Although many complex networks are usually considered as distinct objects, they are often a part of larger complex systems, where a number of coexisting topologies interact and depend on each other. For instance, in the Internet, a graph formed by an application (such as WWW or Peer To Peer) is mapped onto the IP topology that, in turn, is mapped on a physical mesh of cables and optical fibers. The topology at every layer is different. Similarly, it is convenient to view a transportation network as a two-layer system, with a network of traffic demands mapped onto the physical infrastructure.

In order to better describe and analyze such systems, we propose a general *multi-layer model*. Here we focus on two layers; we call a *physical graph* $G^\phi = (V^\phi, E^\phi)$ and a *logical graph* $G^\lambda = (V^\lambda, E^\lambda)$ the topologies at the lower and the upper layer, respectively. Every logical edge e^λ is mapped on the physical graph as a physical path $M(e^\lambda) \subset G^\phi$. The set of paths corresponding to all logical edges is called *mapping* $M(E^\lambda)$ of the logical topology on the physical topology. A simple example illustrating these definitions is given in Fig. 1a.

We apply this two-layer framework to study transporta-

tion networks. The undirected, unweighted physical graph G^ϕ captures the physical infrastructure of a transportation network, and the logical graph G^λ reflects the undirected traffic flows. We apply the algorithm described in [3] to extract both layers and the mapping from timetables of public transportation systems. The examples we choose for our study cover three different scales: city (Warsaw, Poland), country (Switzerland), and continent (central Europe, see Fig. 1b). The sizes of the studied topologies range from 1500 nodes in Warsaw to more than 6000 nodes in Europe.

2. THE BIAS OF LOAD ESTIMATORS

In our model we define the *load* l of a node v^ϕ as the sum of the weights of all logical edges whose mapping traverse v^ϕ in the physical graph:

$$l(v^\phi) = \sum_{e^\lambda: v^\phi \in M(e^\lambda)} w(e^\lambda) \quad (1)$$

In a transportation network $l(v^\phi)$ is the total amount of traffic that flows through the node v^ϕ .

For comparison purposes, we present below two *load estimators* based exclusively on the topology of the physical graph G^ϕ . Our first metric is *node degree* k^ϕ . It seems natural that the nodes with a high degree carry more traffic than the less connected nodes. Our second metric is *betweenness* b^ϕ [2]. Betweenness aims at capturing the amount of information passing through a vertex. Indeed, many authors take betweenness directly as a measure of load (see references in [4]).

In [4] we compare the geographical distribution of the real load, and its two estimators, node degree and betweenness. We find that the correlation between the resulting values is very low. This leads us to the following question: Why do the load estimators fail to mimic the real load pattern? As the ways of constructing the load metrics vary greatly, it is seems difficult to speak about the origins of these differences in a systematic way. However, both studied load estimators can be *recast in a two-layer view* as a specific, ‘load generating’ logical graph mapped onto the physical graph G^ϕ (the same for all). The fundamental properties of the load metrics are then captured by their logical graphs, providing us with a common ground for a systematic comparison. We find that these graphs differ greatly in a number of basic network characteristics, such as planarity, density, degree and weight distribution or edge length distribution. We schematically compare them in Fig. 1c. We conclude that the logical graphs belong to completely different classes, and therefore the three studied load metrics are inherently different.

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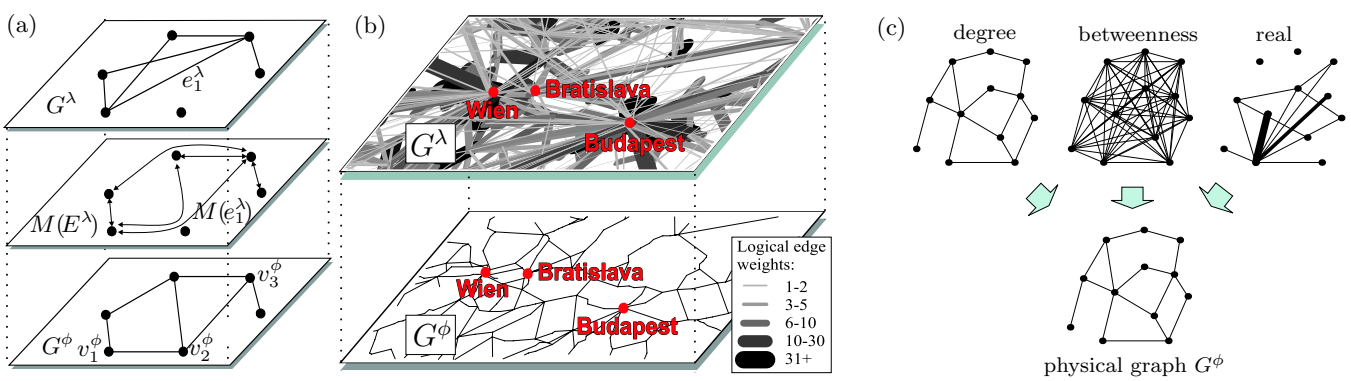


Figure 1: (a) An illustration of the two-layer model with the actual mapping $M(E^\lambda)$ of the logical graph G^λ on the physical graph G^ϕ . The logical edge e_1^λ is mapped on G^ϕ as the path $M(e_1^\lambda) = (v_1^\phi, v_2^\phi, v_3^\phi)$. (b) Two layers in a transportation network (here we show a fragment of the European railway data set). The nodes are the train stations. The edges in the physical graph are the existing rail tracks connecting neighboring stations. Every edge in the logical graph connects the first and the last station of a particular train route; its weight reflects the number of trains following this route in any direction. (c) Three logical ‘load generating’ graphs that, when mapped with shortest paths on the physical graph G^ϕ , reproduce the three load metrics: node degree (left), betweenness (middle) and real load (right).

3. ROBUSTNESS OF LAYERED COMPLEX NETWORKS

A layered perspective may completely change our view on the error and attack tolerance [1] of many complex systems. For example, a failure of a single physical edge affects all logical edges that are mapped on it. If the paths defined by the mapping cannot be changed then every affected logical edge is (temporarily) eliminated. In the context of railway networks, this would mean a cancellation of all trains traversing the faulty rail track. Consequently, a tiny, seemingly unharmed (from a one-layer perspective) disruption of the physical graph might destroy a substantial part of the logical graph, rendering the whole system useless in practice.

As an illustration of this phenomenon, in Fig. 2 we present the results of a simulation of an attack on the most loaded physical edges in the European railway system. The upper curve represents the largest component size in the physical graph, which is a classic metric of network robustness [1]. According to this measure the network appears to be robust to the attack. However, at the same time, the fraction of affected and deleted logical links (traffic flows) drops dramatically. In particular, a deletion of only 3% of the physical edges results in breaking more than 50% of the train connections! This means that the service provided by the network is reduced by half, although the network connectivity is hardly affected (largest component size is equal to about 95% of the original size).

We conclude that there is a need for new robustness measures that capture not only purely topological aspects, but also the functionality of the system as a whole. The measure we propose (number of affected and remaining logical links) is a step in this direction.

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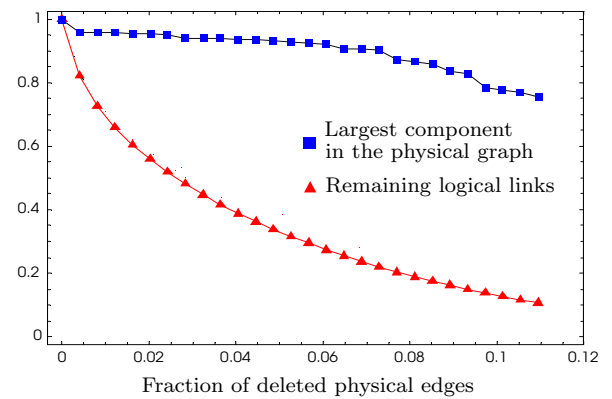


Figure 2: Robustness of the European railway network from two-layer perspective. We attack the most loaded physical edges and observe the size of the largest connected physical component (squares) and fraction of remaining logical links (triangles).

4. REFERENCES

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