Structural Robustness of Complex networks

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ABSTRACT

Researchers have been analyzing network robustness by two approaches, namely network modelling and network analysis. However, there are no consensuses on the use of the network metrics. This paper studies two commonly used network metrics on the study of network robustness, namely average shortest path and diameter, and tries to develop a general principle to measure the network robustness. A metro network, namely Newcastle Metro network, is studied. This paper summarized that the network should be measured by both network disconnectedness and the pre-existing network metric such as average shortest path and diameter to investigate the impact of the loss of a link in the network.

1. INTRODUCTION

Researchers have been analyzing network robustness by two approaches, namely network modelling (e.g. [1][2][3]) and network analysis (e.g. [4][5][6]). Network modelling is to build a mathematically defined network and then measured by network metrics. Network analysis is to measure the existing real-world networks by applying network metrics. Researchers attempted to simulate different scenarios of disruptions on the networks and measure theirs impacts.

Both approaches use network metrics extensively to capture the impact of the simulated disruptions. However, there are no consensuses on the use of the network metrics. This paper studies two commonly used network metrics on the study of network robustness and tries to develop a general principle to measure the network robustness. A metro network, namely Newcastle Metro network, is studied.

2. SCENARIO-BASED ANALYSIS

In the past works, researchers often analyzed the network along the vertical dimensions, i.e. varying the degree of disruptions to identify the responses of the network. Analyzing a network along vertical dimensions makes an underlying assumption that the network consists of homogenous nodes and links. However, real-world networks are heterogeneous and unbalanced which each link has different importance. Hence, each link poses different impacts to the network. This paper proposes a scenario-based analysis that investigates the horizontal dimensions of the network, i.e. attacking different links individually, to understand the importance of individual links in the network.

The impact and occurrences of network disruptions was obtained by simulation. This simulation removed a link, measured the disrupted network and restored the link. The effect of the disruptions was measured by connectivity, average shortest path, diameter, and network disconnectedness. When all the links have undergone this process, a set of data on network disruptions were obtained. A comparison of the pre-existing network metrics and the proposed network metrics are drawn to Newcastle Metro network.

3. AVERAGE SHORTEST PATH

Average Shortest Path is the average of the shortest distance between every pairs of nodes. Equation 1 shows the definition of average shortest path.

$$ASP = \frac{1}{n(n-1)} \sum_i \sum_j d_{ij} \tag{1}$$

where d_{ij} is the shortest path in the network for all nodes from i to j and n is the number of nodes in the network

Average shortest path is a typical network statistic to measure the network distance of a distributed network. Given that the network is connected, the average shortest path is greater when the network undergoes disruptions. As some of the links or nodes are lost or deleted, and some of these nodes or links may form part of the shortest path in the network before disruptions, the shortest path may be longer due to the deletion of these nodes or links.

However, if the disruptions of the network are so severe that the network is fragmented, the shortest paths between the fragmented nodes and other nodes are infinity. Then the average shortest path becomes infinity. In this situation, the average shortest path can only show that the network is fragmented and cannot show the degree of fragmentation.

Figure 1 shows the average shortest path of Newcastle Metro network. In the study, the effect of 58 possible deletions of a single link on the network is measured. These scenarios were plotted along the X-axis and sequenced in the decreasing order of disconnectedness (See explanation in Section 5).

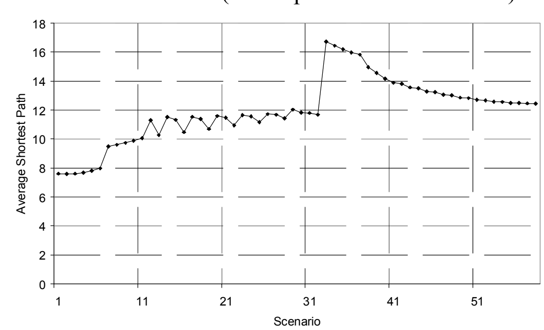

Figure 1 Average shortest path of Newcastle Metro network

The average shortest path shows two different trends depending on whether it is either fragmented or connected. When the network is fragmented, the average shortest path is smaller than that of the connected network because the network is reduced in size due to fragmentation. The general trend of the network metric is that the higher the average shortest path, the better is the connection of the network. In contrast, when the network is connected, the general trend of the network metric is that the smaller the average shortest path, the closer the distance between nodes.

4. DIAMETER

Diameter is defined as the longest shortest path between any pair of nodes of a distributed network (Equation 2).

$$Dia = Max(d_{ij}) \tag{2}$$

where d_{ij} is the shortest path between node *i* and node *j*

When a network is disrupted, the diameter will become larger. Some of the links or nodes that are lost or deleted may constitute part of the shortest path in the network, so the shortest path after disruptions may be longer as these nodes or links are deleted. However, if the disruptions of the network are severe enough, the network may become fragmented into more than one component. Then the diameter will become infinite.

Figure 2 shows the diameter of Newcastle Metro network. In the study, the effect of 58 possible deletions of a single link on the network is measured. These scenarios were plotted along the X-axis and sequenced in the decreasing order of disconnectedness (See Section 5 for explanation).

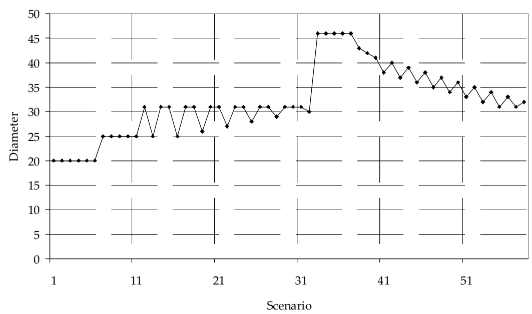

Figure 2 Diameter of Newcastle Metro network

The diameter shows two different trends depending on whether it is either fragmented or connected, the diameter is smaller than that of the connected network because the network is reduced in size due to fragmentation. The general trend of the network metric is that the larger the diameter, the better is the connection of the network. In contrast, when the network is connected, the general trend of the network metric is that the smaller the diameter, the closer the distance between nodes, and hence, the better is the performance of the network.

5. NETWORK DISCONNECTEDNESS

Network disconnection is the proportion of unreachable pairs compared to the theoretical maximum of a completely connected network. Equation 3 shows the definition of network disconnectedness.

$$NetDis = \frac{\varepsilon}{l_{\max}} \tag{3}$$

where N is the fragmented network, ε is the number of unreachable pair of nodes which equals the number of fragmented pairs multiplied by the number of remaining nodes and $l_{\max}$ is the maximal possible number of links.

When nodes are disconnected from the network, the network disconnectedness shows the impact of the losses of a single link or node to the entire network. Network disconnectedness gives the fraction of fragmented nodes due to the disconnection of a single link.

In the study, the effect of 58 possible deletions of a single link on the network is measured. These scenarios were plotted along the X-axis and sequenced in the decreasing order of disconnectedness. Figure 3 shows the network disconnectedness of Newcastle Metro network.

The network disconnectedness measured the fragmentation of the network. When the network is fragmented, the smaller the network disconnectedness, the better the network performs. The network disconnectedness drops to zero, since the network is connected.

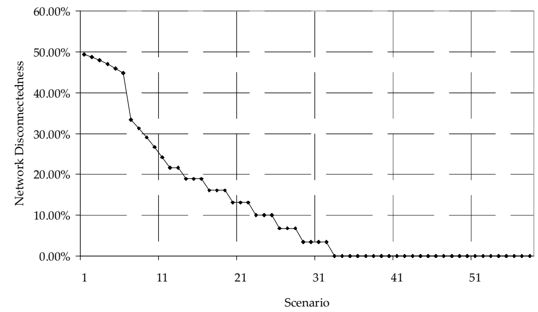

Figure 3 Network Disconnectedness

6. CONCLUSION

Sections 3 and 4 showed that the pre-existing metrics is unable to distinguish between fragmented and connected network. Section 5 showed that the proposed metric is capable to distinguish between fragmented and connected network by measuring the network disconnectedness. This suggests that a combination of metrics should be used to measure the whole spectrum of the network, i.e. from fragmented networks to connected networks.

This paper summarized that the network should be measured by both network disconnectedness and the pre-existing network metric such as average shortest path and diameter to investigate the impact of the loss of a link in the network. The network should be first measured by the network disconnectedness. The network disconnectedness distinguishes networks between fragmented and connected and evaluates the network connectivity. The network should then be measured by the pre-existing metric such as average shortest path. The average shortest path evaluates the distance between nodes. The combination of these metrics provides a comprehensive measure to study the effect of network disruptions. A further research should be conducted to examine the combined use of these metrics.

7. REFERENCES

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