

# Network-based Ranking System for U.S. College Football \*

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## ABSTRACT

American college football faces a conflict created by the desire to stage national championship games between the best teams of a season when there is no conventional playoff system to decide which those teams are. Instead, ranking of teams is based on their record of wins and losses during the season, but each team plays only a small fraction of eligible opponents, making the system underdetermined or contradictory or both. It is an interesting challenge to create a ranking system that at once is mathematically well-founded, gives results in general accord with received wisdom concerning the relative strengths of the teams, and is based upon intuitive principles, allowing it to be accepted readily by fans and experts alike. Here we introduce a one-parameter ranking method that satisfies all of these requirements and is based on a network representation of college football schedules.

## Keywords

Schedule network, sports ranking, centrality, probability

## 1. INTRODUCTION

Inter-university competition in (American) football is big business in the United States. Games are televised on national TV; audiences number in the millions and advertising revenues in the hundreds of millions (of US dollars). Strangely, however, there is no official national championship in college football, despite loudly-voiced public demand for such a thing. In other sports, such as soccer or basketball, there are knockout competitions in which schedules of games are drawn up in such a way that at the end of the competition there is an undisputed “best” team—the only team in the league that remains unbeaten. A simple pairwise elimination tournament is the most common scheme.

The difficulty with college football is that games are mostly played in **conferences**, which are groups of a dozen or so colleges chosen on roughly geographic grounds. In a typical season about 75% of games are played between teams belonging to the same conference. As a result there is normally an undisputed champion for each individual conference, but not enough games are played between conferences to allow an overall champion to be chosen unambiguously.

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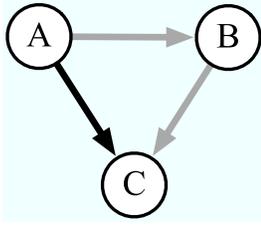
Some other sports also use the conference system, and in those sports an overall champion is usually chosen via a separate knockout tournament organized among the winners and runners up in the individual conferences. In college football, however, for historical and other reasons, there is no such post-season tournament.

To fulfill the wishes of the fans for a national championship, several of the major conferences have adopted a system called the Bowl Championship Series (BCS), in which one of four existing post-season “bowl games” — the Rose, Sugar, Fiesta, and Orange Bowls—is designated the national championship game on a rotating basis and is supposed to match the top two teams of the regular season [3]. (A new system has been adopted for the upcoming 2006 season: the BCS will host a “BCS Championship Game”, bringing the number of BCS bowl games to five.) The problem is how to decide which the top teams are. One can immediately imagine many difficulties. Simply choosing unbeaten teams will not necessarily work: what if there are more than two, or only one, or none? How should one account for teams that play different numbers of regular-season games, and for “strength of schedule”—the fact that some teams by chance inevitably play against tougher opponents than others? What about margins of victory? Should a decisive victory against your opponent count for more than a narrow victory? Should home games be counted differently from away games? [1, 2, 4, 5, 6, 7, 9]

Currently football teams are ranked using a weighted composite score called the BCS ranking that combines a number of these methods with polls of knowledgeable human judges. The formula used changes slightly from year to year; the most recent version averages six computer algorithms. There is, however, considerable unhappiness about the system and widespread disagreement about how it should be improved<sup>1</sup>. There is, thus, plenty of room for innovation.

In this paper, we present a new method of ranking based on a mathematical formulation that corresponds closely to the types of arguments typically made by sports fans in comparing teams. Our method turns out to be equivalent to a well-known type of centrality measure defined on a di-

<sup>1</sup>It was originally hoped that the BCS rankings would help generate consensus about the true number 1 and 2 teams, resulting in an undisputed national champion, but it hasn’t always worked out that way. Most recently in 2003, for instance, the AP poll awarded its top spot to the University of Southern California, contradicting the overall BCS rankings, which awarded top honors to the Louisiana State University, and resulting in a “split” national title of the kind the system was designed to avoid.



**Figure 1: If team A has beaten team B, and team B has beaten team C, team A scores an indirect win over team C (indicated by the bold arrow).**

rected network representing the pattern of wins and losses in regular-season games.

## 2. THE METHOD

Perhaps the simplest measure of team standing is the win-loss differential, i.e., the number of games a team wins during the season minus the number it loses. (In American football there are no tied games—games are extended until there is a winner.) Indeed, the win-loss differential is almost the only measure that everyone seems to agree upon. It is unfortunate therefore that in practice it correlates rather poorly with expert opinions about which teams are best, for many of the reasons cited in the previous section, such as variation in strength of schedule. As we show here, however, we can correct for these problems, at least in part, by considering not just direct wins and losses, but also indirect ones.

One often hears from sports fans arguments of the form: “Although my team A didn’t play your team C this season, it did beat B who in turn beat C. Therefore A is better than C and would have won had they played a game.” (See Fig. 1.) In fact, the argument is usually articulated with less clarity than this and more beer, but nonetheless we feel that the general line of reasoning has merit. What the fan is saying is that, in addition to a real, physical win (loss) against an opponent, an *indirect win (loss)* of the type described should also be considered indicative of a team’s strength (weakness).

If we represent the game schedule as a directed network where an arrows runs from the winner to the loser of a game, indirect losses and wins, as defined above, correspond to *directed paths of length 2* in the network, to and from the team.

A particularly nice property of these indirect wins is that a direct win against a strong opponent—a team that has itself won many games—is highly rewarding, giving you automatically a large number of indirect wins. Thus, when measured in terms of indirect wins, the ranking of a team automatically allows for the strength of schedule.

And there is no need to stop here: one can consider higher-order indirect wins (or losses) of the form A beats B beats C beats D, and so forth. These correspond to directed paths in the network of length three or more. Our proposed ranking scheme counts indirect wins and losses at all distances in the network, but those at greater distances count for less, because we feel it natural that a direct win against a team should count for more than the mere supposed victory of an indirect win.

Mathematically, we can express these ideas in terms of the adjacency matrix  $\mathbf{A}$  of the network, an  $n \times n$  real asym-

metric matrix, where  $n$  is the number of teams, and its element  $A_{ij}$  is equal to the number of times team  $j$  has beaten team  $i$  (usually 0 or 1, but occasionally 2). We define the *win scores*  $\mathbf{w} = (w_1, w_2, \dots)$  and *loss scores*  $\mathbf{l} = (l_1, l_2, \dots)$ , the weighted sums of direct and indirect wins or losses, equal to

$$\mathbf{w} = (I - \alpha \mathbf{A}^T)^{-1} \cdot \mathbf{k}^{\text{out}}, \quad \mathbf{l} = (I - \alpha \mathbf{A})^{-1} \cdot \mathbf{k}^{\text{in}}, \quad (1)$$

where  $\mathbf{k}^{\text{out}} = (k_1^{\text{out}}, k_2^{\text{out}}, \dots)$  and  $\mathbf{k}^{\text{in}} = (k_1^{\text{in}}, k_2^{\text{in}}, \dots)$  are the out-degrees and the in-degrees, and  $\alpha$  is a free parameter that determines the relative weight of an indirect win or loss. Finally, we define  $\mathbf{w} - \mathbf{l}$  as the final score of each team.

We also discuss a practical method of choosing the value of  $\alpha$  to use, and compare the resultant rankings with the official BCS rankings from the past several seasons [8].

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