A social growth ensemble for networked agents

Takuya Yamano*
Social Science Research Institute, International Christian University
3-10-2 Osawa, Mitaka
181-8585 Tokyo, Japan
yamano@uni-bremen.de

ABSTRACT

Network effects which exist in social company environments definitely affect indivisual growth processes of companies. We focus on the Gompertz type growth model and provide some analytical results in the case that the Gompertz growth parameters are different by agents. This formulation leads to a construction of a social growth ensemble in statistical mechanics sense.

Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Sociology*

General Terms

Theory

Keywords

Growth model, Gompertz function, Social ensemble

1. INTRODUCTION

Company growth mechanisms and these statistical features can be one of main social network problems in social sciences. The phenomena of growing companies have similarities to biological growth (tomour) developments, to some extent, in that agents interact according to availabilities of resources. Traditionally, the Gompertz function [4] is known to fit tomour growth data [6, 7, 2] but the origin remains unclear [3, 5]. The tomour growth can be seen as a consequence of the complex interactions (often networked chemical reactions) of the cells in tomours and of the competition for the limited resources (nutrients). In this sense, the tumour growth is similar to the company growth in a heterogeneous society because companies grow (sometimes shrink) in an environment of a limited capital resources and interact each other.

Growth rates of companies are different each other. Some companies may be in a phase of a rapid growth due to the success of the market strategy. On the other hand, some companies may have a stagnation in profits in the same periods. Identifying the relation between dynamically changing network structures of companies and a growth rate of the individual companies is a quite tough problem.

In stead of using approaches by means of differencial equations by which each companies are assumed to follow, we focus on an alternative approach based on a statistical ensemble of companies whose growth rates are different in general due to interactions among them. In this contribution, we use the Gompertz function and aim at obtaining some analytical expressions to see the implications for real company growth in societies.

2. GROWTH FUNCTION AND PROPERTIES

The Gompertz function we use is of the form, $f(t) = f(0)e^{a(1-e^{-bt})}$, where $f(t)$ denotes the number of companies and $f(0)$ is an initial number of companies. a and b are constants. We show that the time fluctuation with a common growth rate b is found to be decay as b^{-2} if we use a velocity of growth for the averaging function. For a set of the different growth rates b's, its fluctuation can be expressed by the generalized hypergeometric function and the behavior is shown as a function of time in the case that we trancate the expansion of the obtained expression up to $O(a^4)$.

To understand the Gompertz type growth curve well, we show that a type of process $F(t)$ provides the Bose-Einstein statistics form if the growth factor takes the Gompertzian when $bt << 1$ and the Fermi-Dirac statistics ones if the process has negative feedbacks. The former case can be understood by the Euler's transform between original growth curve and new process $F(t)$ [1] but the latter case is not known. This implies that in very early stages or when the growth rate is small, the company growth can be compatible with statistical mechanics ground. We also revisit the entropy of the Gompertz form of growths [8].

3. SUMMARY

We report in this contribution that a set of companies with different growth rates construct ensemble which reflects the network structures.

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