Processes Taking Place on Networks

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- Traffic flow on roads
- Airline flights between airports
- Data traffic on the Internet
- Spread of diseases between people
- Spread of computer viruses between computers
- Spread of news or rumors over social networks
- Reactions in metabolic networks
- Energy flow in food webs
Example: A computer virus
FIGURE 2. Probable cases of severe acute respiratory syndrome, by reported source of infection* — Singapore, February 25–April 30, 2003

* Patient 1 represents Case 1; Patient 6, Case 2; Patient 35, Case 3; Patient 130, Case 4; and Patient 127, Case 5. Excludes 22 cases with either no or poorly defined direct contacts or who were cases translocated to Singapore and the seven contacts of one of these cases.
Epidemiology

- As a disease spreads through a population, individuals pass through different states
- In the simplest case there are two states:
  - Susceptible (S) means you don't have the disease
  - Infective (I) means you do and *you can pass it on*
• We can add more states:
  - Recovered (R) means you have had the disease, gotten over it, and *are now immune*
  - Exposed (E) means you have caught the disease, but are *not yet infective* (though you will be)
- We can also change the flow-chart

- Individuals who have recovered can lose their immunity and become susceptible again:
Traditional approaches

• Traditionally the way to model diseases was to make the full mixing assumption

  - A susceptible individual catches the disease from an infective one

  - The probability of transmission between any two individuals in given compartments is the same for all individuals in those compartments

• This is obviously wrong, but it's convenient because it makes the math easier
Traditional approaches

- For example: the simple SIR model is then governed by the equations:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta \frac{SI}{n}, \\
\frac{dI}{dt} &= \beta \frac{SI}{n} - \gamma I, \\
\frac{dR}{dt} &= \gamma I.
\end{align*}
\]
Network approaches

• Once we bring networks into the picture, the differential equation approach breaks down. We need other techniques

• One crucial observation is that diseases are more likely to find people with many connections:
Number of secondary cases is $T(k-1)$, where $T$ is the transmissibility:

\[ R_0 = T \frac{\sum_i k_i (k_i - 1)}{\sum_i k_i} = T \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} \]

Disease spreads if $R_0 > 1$ or if

\[ T > \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle} \]
• For networks with highly skewed degree distributions, \( \langle k^2 \rangle \) can be very large, making \( T \) very small.

• For power-law degree distributions \( \langle k^2 \rangle \) can vanish entirely in the limit of large network size, meaning \( T = 0 \) and the disease always spreads.

• We need to be careful however. This result can be modified by other things such as
  - Correlations in the network
  - Local network structure

• Let us look at some of the details...
Percolation theory
Example: Competing pathogens
Example: Assortative mixing
Assortativity by degree

Maslov et al. 2004
Assortativity by degree

Pastor-Satorras and Vespignani 2001
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 assortative  

 disassortative